Vector Algebra

Question1

Among the given pair of vectors, the resultant of two vectors can never be 3 units. The vectors are

KCET 2024

Options:

A. 1 unit and 2 units

B. 3 units and 6 units

C. 2 units and 5 units

D. 4 units and 8 units

Answer: D

Solution:

To determine in which case the resultant of two vectors can never be 3 units, we need to use the triangle inequality theorem for vectors:

For two vectors with magnitudes $|\mathbf{A}|$ and $|\mathbf{B}|$, the resultant vector $|\mathbf{R}|$ satisfies:

$||\mathbf{A}| - |\mathbf{B}|| \leq |\mathbf{R}| \leq |\mathbf{A}| + |\mathbf{B}|$

Let's analyze each option to see if a resultant of 3 units is possible:

Option A: 1 unit and 2 units

Minimum resultant: $|1 - 2| = 1$

Maximum resultant: $1 + 2 = 3$

A resultant of 3 is possible.

Option B: 3 units and 6 units

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Minimum resultant: $|3 - 6| = 3$

Maximum resultant: $3 + 6 = 9$

A resultant of 3 is possible.

Option C: 2 units and 5 units

Minimum resultant: $|2 - 5| = 3$

Maximum resultant: $2 + 5 = 7$

A resultant of 3 is possible.

Option D: 4 units and 8 units

Minimum resultant: $|4 - 8| = 4$

Maximum resultant: $4 + 8 = 12$

A resultant of 3 units is not possible as the minimum resultant is 4.

Therefore, the pair of vectors where the resultant can never be 3 units is **Option D (4 units and 8 units)**.

Question2

If $\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$ is perpendicular to $\mathbf{B} = 4\hat{\mathbf{j}} - 4\hat{\mathbf{i}} + \alpha\hat{\mathbf{k}}$, then the value of α is

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Options:

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. 1

D. -1

Answer: A

Solution:

Given the vectors:

$\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$

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$\mathbf{B} = 4\hat{\mathbf{j}} - 4\hat{\mathbf{i}} + \alpha\hat{\mathbf{k}}$

$\mathbf{B} = -4\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \alpha\hat{\mathbf{k}}$

Since $\mathbf{A}$ is perpendicular to $\mathbf{B}$, their dot product is zero:

$\mathbf{A} \cdot \mathbf{B} = 0$

This translates to:

$(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 8\hat{\mathbf{k}}) \cdot (-4\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \alpha\hat{\mathbf{k}}) = 0$

Calculating the dot product gives:

$2(-4) + 3(4) + 8(\alpha) = 0$

Simplifying the equation:

$-8 + 12 + 8\alpha = 0$

$8\alpha + 4 = 0$

$8\alpha = -4$

$\alpha = -\frac{4}{8} = -\frac{1}{2}$

Therefore, the value of α is $-\frac{1}{2}$.

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